CHAPTER 4

THE UNCERTAINTY PRINCIPLE

The success of scientific theories, particularly Newton's theory of gravity, led the French scientist the Marquis de Laplace at the beginning of the nineteenth century to argue that the universe was completely deterministic. Laplace suggested that there should be a set of scientific laws that would allow us to predict everything that would happen in the universe, if only we knew the complete state of the universe at one time. For example, if we knew the positions and speeds of the sun and the planets at one time, then we could use Newton's laws to calculate the state of the Solar System at any other time. Determinism seems fairly obvious in this case, but Laplace went further to assume that there were similar laws governing everything else, including human behavior.

The doctrine of scientific determinism was strongly resisted by many people, who felt that it infringed God's freedom to intervene in the world, but it remained the standard assumption of science until the early years of this century. One of the first indications that this belief would have to be abandoned came when calculations by the British scientists Lord Rayleigh and Sir James Jeans suggested that a hot object, or body, such as a star, must radiate energy at an infinite rate. According to the laws we believed at the time, a hot body ought to give off electromagnetic waves (such as radio waves, visible light, or X rays) equally at all frequencies. For example, a hot body should radiate the same amount of energy in waves with frequencies between one and two million million waves a second as in waves with frequencies between two and three million million waves a second. Now since the number of waves a second is unlimited, this would mean that the total energy radiated would be infinite.

In order to avoid this obviously ridiculous result, the German scientist Max Planck suggested in 1900 that light, X rays, and other waves could not be emitted at an arbitrary rate, but only in certain packets that he called quanta. Moreover, each quantum had a certain amount of energy that was greater the higher the frequency of the waves, so at a high enough frequency the emission of a single quantum would require more energy than was available. Thus the radiation at high frequencies would be reduced, and so the rate at which the body lost energy would be finite.

The guantum hypothesis explained the observed rate of emission of radiation from hot bodies very well, but its implications for determinism were not realized until 1926, when another German scientist, Werner Heisenberg, formulated his famous uncertainty principle. In order to predict the future position and velocity of a particle, one has to be able to measure its present position and velocity accurately. The obvious way to do this is to shine light on the particle. Some of the waves of light will be scattered by the particle and this will indicate its position. However, one will not be able to determine the position of the particle more accurately than the distance between the wave crests of light, so one needs to use light of a short wavelength in order to measure the position of the particle precisely. Now, by Planck's quantum hypothesis, one cannot use an arbitrarily small amount of light; one has to use at least one quantum. This quantum will disturb the particle and change its velocity in a way that cannot be predicted. moreover, the more accurately one measures the position, the shorter the wavelength of the light that one needs and hence the higher the energy of a single quantum. So the velocity of the particle will be disturbed by a larger amount. In other words, the more accurately you try to measure the position of the particle, the less accurately you can measure its speed, and vice versa. Heisenberg showed that the uncertainty in the position of the particle times the uncertainty in its velocity times the mass of the particle can never be smaller than a certain quantity, which is known as Planck's constant. Moreover, this limit does not depend on the way in which one tries to measure the position or velocity of the particle, or on the type of particle: Heisenberg's uncertainty principle is a fundamental, inescapable property of the world.

The uncertainty principle had profound implications for the way in which we view the world. Even after more than seventy years they have not been fully appreciated by many philosophers, and are still the subject of much controversy. The uncertainty principle signaled an end to Laplace's dream of a theory of science, a model of the universe that would be completely deterministic: one certainly cannot predict future events exactly if one cannot even measure the present state of the universe precisely! We could still imagine that there is a set of laws that determine events completely for some supernatural being, who could observe the present state of the universe without disturbing it. However, such models of the universe are not of much interest to us ordinary mortals. It seems better to employ the principle of economy known as Occam's razor and cut out all the features of the theory that cannot be observed. This approach led Heisenberg, Erwin Schrodinger, and Paul Dirac in the 1920s to reformulate mechanics into a new theory called quantum mechanics, based on the uncertainty principle. In this theory particles no longer had separate, well-defined positions and velocities that could not be observed, Instead, they had a quantum state, which was a combination of position and velocity.

In general, quantum mechanics does not predict a single definite result for an observation. Instead, it predicts a number of different possible outcomes and tells us how likely each of these is. That is to say, if one made the same measurement on a large number of similar systems, each of which started off in the same way, one would find that the result of the

A Brief History of Time - Stephen Hawking... Chapter 4

measurement would be A in a certain number of cases, B in a different number, and so on. One could predict the approximate number of times that the result would be A or B, but one could not predict the specific result of an individual measurement. Quantum mechanics therefore introduces an unavoidable element of unpredictability or randomness into science. Einstein objected to this very strongly, despite the important role he had played in the development of these ideas. Einstein was awarded the Nobel Prize for his contribution to quantum theory. Nevertheless, Einstein never accepted that the universe was governed by chance; his feelings were summed up in his famous statement "God does not play dice." Most other scientists, however, were willing to accept quantum mechanics because it agreed perfectly with experiment. Indeed, it has been an outstandingly successful theory and underlies nearly all of modern science and technology. It governs the behavior of transistors and integrated circuits, which are the essential components of electronic devices such as televisions and computers, and is also the basis of modern chemistry and biology. The only areas of physical science into which quantum mechanics has not yet been properly incorporated are gravity and the large-scale structure of the universe.

Although light is made up of waves, Planck's quantum hypothesis tells us that in some ways it behaves as if it were composed of particles: it can be emitted or absorbed only in packets, or quanta. Equally, Heisenberg's uncertainty principle implies that particles behave in some respects like waves: they do not have a definite position but are "smeared out" with a certain probability distribution. The theory of quantum mechanics is based on an entirely new type of mathematics that no longer describes the real world in terms of particles and waves; it is only the observations of the world that may be described in those terms. There is thus a duality between waves and particles in quantum mechanics: for some purposes it is helpful to think of particles as waves and for other purposes it is better to think of waves as particles. An important consequence of this is that one can observe what is called interference between two sets of waves or particles. That is to say, the crests of one set of waves may coincide with the troughs of the other set. The two sets of waves then cancel each other out rather than adding up to a stronger wave as one might expect Figure 4:1.





A familiar example of interference in the case of light is the colors that are often seen in soap bubbles. These are caused by reflection of light from the two sides of the thin film of water forming the bubble. White light consists of light waves of all different wavelengths, or colors, For certain wavelengths the crests of the waves reflected from one side of the soap film coincide with the troughs reflected from the other side. The colors corresponding to these wavelengths are absent from the reflected light, which therefore appears to be colored. Interference can also occur for particles, because of the duality introduced by quantum mechanics. A famous example is the so-called two-slit experiment Figure 4:2.

A Brief History of Time - Stephen Hawking... Chapter 4





Consider a partition with two narrow parallel slits in it. On one side of the partition one places a source of fight of a particular color (that is, of a particular wavelength). Most of the light will hit the partition, but a small amount will go through the slits. Now suppose one places a screen on the far side of the partition from the light. Any point on the screen will receive waves from the two slits. However, in general, the distance the light has to travel from the source to the screen via the two slits will be different. This will mean that the waves from the slits will not be in phase with each other when they arrive at the screen: in some places the waves will cancel each other out, and in others they will reinforce each other. The result is a characteristic pattern of light and dark fringes.

The remarkable thing is that one gets exactly the same kind of fringes if one replaces the source of light by a source of particles such as electrons with a definite speed (this means that the corresponding waves have a definite length). It seems the more peculiar because if one only has one slit, one does not get any fringes, just a uniform distribution of electrons across the screen. One might therefore think that opening another slit would just increase the number of electrons hitting each point of the screen, but, because of interference, it actually decreases it in some places. If electrons are sent through the slits one at a time, one would expect each to pass through one slit or the other, and so behave just as if the slit it passed through were the only one there – giving a uniform distribution on the screen. In reality, however, even when the electrons are sent one at a time, the fringes still appear. Each electron, therefore, must be

A Brief History of Time - Stephen Hawking... Chapter 4

passing through both slits at the same time!

The phenomenon of interference between particles has been crucial to our understanding of the structure of atoms, the basic units of chemistry and biology and the building blocks out of which we, and everything around us, are made. At the beginning of this century it was thought that atoms were rather like the planets orbiting the sun, with electrons (particles of negative electricity) orbiting around a central nucleus, which carried positive electricity. The attraction between the positive and negative electricity was supposed to keep the electrons in their orbits in the same way that the gravitational attraction between the sun and the planets keeps the planets in their orbits. The trouble with this was that the laws of mechanics and electricity, before quantum mechanics, predicted that the electrons would lose energy and so spiral inward until they collided with the nucleus. This would mean that the atom, and indeed all matter, should rapidly collapse to a state of very high density. A partial solution to this problem was found by the Danish scientist Niels Bohr in 1913. He suggested that maybe the electrons were not able to orbit at just any distance from the central nucleus but only at certain specified distances. If one also supposed that only one or two electrons could orbit at any one of these distances, this would solve the problem of the collapse of the atom, because the electrons could not spiral in any farther than to fill up the orbits with e least distances and energies.

This model explained quite well the structure of the simplest atom, hydrogen, which has only one electron orbiting around the nucleus. But it was not clear how one ought to extend it to more complicated atoms. Moreover, the idea of a limited set of allowed orbits seemed very arbitrary. The new theory of quantum mechanics resolved this difficulty. It revealed that an electron orbiting around the nucleus could be thought of as a wave, with a wavelength that depended on its velocity. For certain orbits, the length of the orbit would correspond to a whole number (as opposed to a fractional number) of wavelengths of the electron. For these orbits the wave crest would be in the same position each time round, so the waves would add up: these orbits would correspond to Bohr's allowed orbits. However, for orbits whose lengths were not a whole number of wavelengths, each wave crest would eventually be canceled out by a trough as the electrons went round; these orbits would not be allowed.

A nice way of visualizing the wave/particle duality is the so-called sum over histories introduced by the American scientist Richard Feynman. In this approach the particle is not supposed to have a single history or path in space-time, as it would in a classical, nonquantum theory. Instead it is supposed to go from A to B by every possible path. With each path there are associated a couple of numbers: one represents the size of a wave and the other represents the position in the cycle (i.e., whether it is at a crest or a trough). The probability of going from A to B is found by adding up the waves for all the paths. In general, if one compares a set of neighboring paths, the phases or positions in the cycle will differ greatly. This means that the waves associated with these paths will almost exactly cancel each other out. However, for some sets of neighboring paths the phase will not vary much between paths. The waves for these paths will not cancel out Such paths correspond to Bohr's allowed orbits.

With these ideas, in concrete mathematical form, it was relatively straightforward to calculate the allowed orbits in more complicated atoms and even in molecules, which are made up of a number of atoms held together by electrons in orbits that go round more than one nucleus. Since the structure of molecules and their reactions with each other underlie all of chemistry and biology, quantum mechanics allows us in principle to predict nearly everything we see around us, within the limits set by the uncertainty principle. (In practice, however, the calculations required for systems containing more than a few electrons are so complicated that we cannot do them.)

Einstein's general theory of relativity seems to govern the large-scale structure of the universe. It is what is called a classical theory; that is, it does not take account of the uncertainty principle of quantum mechanics, as it should for consistency with other theories. The reason that this does not lead to any discrepancy with observation is that all the gravitational fields that we normally experience are very weak. How-ever, the singularity theorems discussed earlier indicate that the gravitational field should get very strong in at least two situations, black holes and the big bang. In such strong fields the effects of quantum mechanics should be important. Thus, in a sense, classical general relativity, by predicting points of infinite density, predicts its own downfall, just as classical (that is, nonquantum) mechanics predicted its downfall by suggesting that atoms should collapse to infinite density. We do not yet have a complete consistent theory that unifies general relativity and quantum mechanics, but we do know a number of the features it should have. The consequences that these would have for black holes and the big bang will be described in later chapters. For the moment, however, we shall turn to the recent attempts to bring together our understanding of the other forces of nature into a single, unified quantum theory.

